LIFETIME INCOME VARIABILITY AND INCOME PROFILES

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1. Introduction

Two types of dynamics provide the macroeconomic context that generates individual incomes. 1) Individual skills, energy, and human capital imply systematic variation in earnings as individuals age. 2) Changes in the price level, productivity, and factor endowments imply global changes in the yield of human, physical, and financial assets held by individuals.

In this study the aggregative effects of both types of dynamics are taken as given, and the proper subject for another investigation. The questions raised by our analysis pertain to the nature and variability of individual income experiences within this global context:

1) Does an individual establish and maintain a relative income position within the birth cohort and population group to which he belongs? Or does income variation reflect random movement relative to the mean experience of the group?

2) Does aging produce increases in the variance of income?

3) Are changes in relative income position randomly distributed in time, or is some simple stochastic process involved?

4) To what extent do the age-income dynamics vary significantly among identifiable population groups?

Considerable interest attaches to these questions. Policy makers would like to know to what extent poverty is a transitory phenomenon. Forecasters would like to know the stability of income as an important factor influencing consumption choices. Tax experts are interested in variability as it affects the need for averaging under a progressive tax law.

In this paper we describe the pattern of income received by individuals filing at least five tax returns in Wisconsin from 1947 to 1959, with observations in at least four pairs of adjacent years. We fit a trend to the pattern observed for each individual. The results may be displayed simply as a distribution of such individual parameters (David and Miller [1970]) or they may be interpreted as observations in a random coefficients model. The latter approach will be developed in the following section. Tests of alternative models are presented in section 3. The paper concludes with a discussion of the relationship between occupational mobility and the model's parameters.

2. Specification of the Model

To normalize income observed for persons of different ages, income of each individual in each year was measured relative to the mean for his birth cohort in that year. Thus the dependent variable in this study is defined

$$y_{it} \equiv \frac{Y_{it}}{C_t(B_i)} \,. \qquad [1]$$

Y_{it} is the adjusted gross income received by i in year t. $C_t(B_i)$ is the mean income for all individuals born within an interval of years that includes the birth year B_i of the individual in question.[1]

The variable y_{it} is invariant to inflation that systematically raises all sources of adjusted gross income. It is also invariant to any change in real return to human, physical, or financial capital that is uniform over the cohort. For example, an improved technology that provides increased return to both capital and labor does not affect y_{it}.

For each birth cohort the expected value of y_{it} is the ratio of expected adjusted gross income for the taxpayer universe to the estimated mean Census income received by the cohort in that year.

Nothing about the choice of y_{it} as a dependent variable imples that a particular age-income profile must apply for any particular group. The global age-income profile for a birth cohort $\{C_t(B_i)\}$ is an average of disparate experiences. Indeed, we may suppose that the population is partitioned into K groups that are homogeneous with respect to human and other capital. It is then reasonable to suppose a common response of incomes within the group to market forces, and a common development of income with aging. We designate that individual i is in the k^{th} group by a prescript; ${}_ky_{it}$ thus refers to the relative income position of the i^{th} individual within the k^{th} group.

Two models are assumed for the generation of ${}_ky_{it}$. The first assumes a systematic trend in relative income position:

$${}_ky_{it} = {}_k\alpha + {}_k\beta(t-1959) + {}_ku_i + {}_kv_{it} \qquad [2]$$

where

$${}_ku_i \sim N(0, {}_k\sigma_u^2) \qquad {}_kv_{it} \sim N(0, {}_k\sigma_v^2) \qquad [3]$$

$$\mathrm{Cov}({}_ku_i, {}_kv_{it}) = 0 \quad i \in k \text{ and } 1948 \le t \le 1959$$

$$\mathrm{Cov}({}_kv_{it}, {}_kv_{it'}) = 0 \quad i \ne i' \in k, \; t \ne t' \text{ and } 1948 \le t, t' \le 1959$$

We refer to this model as the simple trend model.

The parameters ${}_k\alpha$ and ${}_k\beta$ establish a sytematic trend for the adjusted gross income of the k^{th} group relative to the mean income estimated for the cohort. The parameter ${}_k\beta < 0$ indicates that the group loses relative income position during the period observed; ${}_k\beta > 0$ indicates the converse. The parameter ${}_k\alpha$ indicates the expected relative income position for the group in 1959. In addition, individual characteristics determine the distribution of individuals about the group trend according to ${}_ku_i$. ${}_ku_i < 0$ implies that on the average an individual occupies a position below the trend line for the group. Finally, in any given year the relative income position of an individual is determined by a random drawing from the error process that determines ${}_kv_{it}$.

These concepts are illustrated in Figure 1a.

${}_k\alpha$ is given by the ratio ao/bo. A value of ${}_k\beta>0$ is reflected in the larger slope of aa' relative to bb'. Finally cc' reflects $u_i > 0$, where the i^{th} individual has an average relative income position greater than that of the group to which he belongs. The collection of all individual experiences in the group determines the ${}_k\sigma_u$ shown in Figure 1b. The random motion about the trend for the individual is measured relative to the displaced trend observed for the individual shown in the Figure by ${}_k\sigma_v$.

The second model is identical to the first, except that ${}_kw_{it}$ replaces ${}_kv_{it}$ in equation [2]. ${}_kw_{it}$ is defined by the autoregressive relation:

$$\begin{aligned} {}_kw_{it} &= \rho_i\, {}_kw_{i,t-1} + {}_kv_{it} \\ -1 &< \rho_i < 1 \end{aligned} \qquad [4]$$

We refer to this model as the autoregressive model. It will be convenient to distinguish parameters of the autoregressive model by using primes; ${}_k\sigma_u$ refers to the trend model, ${}_k\sigma'_u$ to the autoregressive model. To assure that the variance of ${}_kw_{it}$ is finite, ρ_i must have an absolute value less than unity. If $\rho_i = 1$ the process is a random walk, and estimates of ${}_k\alpha'$, ${}_k\beta'$, ${}_k\sigma'_u$, and ${}_k\sigma'_v$ cannot be obtained from the usual autoregressive transform (Malinvaud [1966] 379-82). We refer to the estimators of the parameters by using corresponding Roman letters ${}_kr_i$, ${}_ka'$, ${}_kb'$, ${}_ks'_u$, ${}_ks'_v$.

Choice between the autoregressive and the simple trend versions of the stochastic model can be made on the basis of the variance explained by the models. We test the hypothesis that estimation of r_i fails to contribute significantly to the explained variance, using the conventional F-test.

Interpretation of the Models

As $\{C_t(B_i)\}$ is the time series of expected income for a cohort, the model provides that the expected income of the k^{th} group is equal to

$$C_t(B_i)({}_k\alpha + {}_k\beta[t - 1959].$$

If the experience of a subpopulation is exactly the same as the cohort taken as a whole ${}_k\alpha = 1$ and ${}_k\beta = 0$.

The model most nearly parallel to our autoregressive model is that of Fase [1970, 10-11]. He develops a stochastic model for the distribution of earnings for individuals of different ages. Upon entry into the labor force at age $\underline{s}$ earnings $\underline{e_s}$ are lognormally distributed according to $\Lambda(e_s; \mu_s, \sigma_s^2)$. Subsequent earnings are autoregressively developed from the drawing obtained at age s:

$$\begin{aligned} \ell n\, e_t &= \ell n\, e_{t-1} + \xi(\tau - t) + u_t \\ t &= s+1,\ s+2,\ \dots\ T \end{aligned} \qquad [5]$$

where u_t is normally distributed and

i) $E(u_t) = 0$

ii) $\mathrm{var}(u_t) = \sigma^2$

iii) $\mathrm{cov}(\ell n\, e_{t-i}, u_t) = 0 \quad i = 1,2,\dots\ t-s$

iv) $\mathrm{cov}(u_t, u_{t+r}) = 0 \quad$ for $t > \sigma$ and $r \neq 0$

For comparison with our autoregressive model, equation [5] can be written as

$${}_ke_{it} = {}_ke_{i,t-1}\ \eta\ \exp\ [-{}_k\xi(t-1959) + {}_ku_{it}]$$

where η is the appropriate constant resulting from the substitution of 1959 for τ.

Fase's model is purley multiplicative and depends on the individual's earnings in the last period. Our autoregressive model is additive in the error terms and depends on cohort income changes as well as individual income changes. Rearranging terms in Appendix equation [A1] gives

$$\rho\ \frac{Y_{i,t-1}}{C_{t-1}(B_i)} + u_i + v_{it}]$$

$$\begin{aligned} Y_{it} = {} & \rho_i Y_{i,t-1}\left(\frac{C_t(B_i)}{C_{t-1}(B_i)}\right) \\ & + (1-\rho_i)\ C_t\ (B_i)({}_k\alpha + {}_k\beta[t-1958] \\ & + C_t(\beta_i)({}_k\beta\rho_i + [1-\rho_i]u_i + v_{it}) \end{aligned} \qquad [7]$$

Aside from the difference between logarithmic and linear formulations, stochastic process [7] is more general than [6] as the formulation permits a weighting of past income experience of the individual (1st term on the r.h.s. of [7]) with the global experiences of the cohort (2nd term on the r.h.s. of [7]).

Both models permit estimation of an initial variance in income positions for the group and a variance associated with the subsequent development of incomes.

3. Empirical Results

Table 1 presents a summary of our two models as fit to individual time series on adjusted gross income for males in Wisconsin for the years 1948-59. Parameters are estimated as appropriately weighted means of individuals' estimates. The justification for this procedure and the formulas used are shown in Balestra and Nerlove [1966, pp. 606-8, equations (43-44)]. The mean intercept estimated exceeds unity for both models. This result is to be expected from the fact that $C_t(B_i)$ was estimated for all income receivers, while the time series data for y_{it} are only available for individuals filing two consecutive tax returns at least four times during the peiod under observation. Persons who file tax returns intermittently because of low incomes will be systematically undersampled in our data. Highly mobile persons who are intermittently out of the state will also be undersampled.

The mean trend is slightly negative. Again this result was to be expected from the discrepancy between the taxpayer universe and the population of income receivers. Young earners, whose incomes are too small to tax, enter the labor force and raise the average cohort income by more than the average annual growth of an individual's income. (See David and Miller [1970], 83-84). A negative trend results for young cohorts.

The autoregressive model is only valid when $-1 < r_i < 1$. Because the models were estimated for individuals and then pooled to obtain ${}_ka', {}_kb'$, some of the r_i were inadmissible.[2] Cases with inadmissible values were excluded from consideration. Individuals for whom $.95 < r_i \leq 1.00$ were also excluded from the tabulations as the estimators for α_i', and β_i' become increasingly unstable as r_i approaches unity. Thus there are 218 individuals for whom we were not able to extract a valid value of the autoregressive coefficient on the basis of an individual's time series. Persons whose time series were excluded reported a shorter time series of data than the remaining population. This is shown in column (2) of Table 1 where the average number of degrees of freedom available for estimating the two models is shown. The autoregressive model entails one more parameter than the simple trend model, and an average of 6.14 degrees of freedom per individual time series are available for parameter estimation. However, individuals with inadmissible r_i were associated with 3.06 degrees of freedom.

A brief consideration of the difference between the simple trend and the autoregressive model suggests the appropriate test to evaluate the contribution of ρ_i. For each individual the autoregressive specification results in one additional regressor. A test of the contribution to the explained variance due to ρ_i, using a standard F-test would be appropriate. For a group of individuals, we can obtain an estimate of contribution to explain variance by pooling and can compare it to the estimate of pooled error variance. The latter can be obtained from line B of table 1.[3] Over the entire population variance explained by r_i fails to exceed what might be expected on the basis of chance ($F=0.945$; $F_{.01}=1.00$). Thus it appears that autoregression, if at all important, is restricted to small population subgroups.

In contrast to this global test for significance of r_i, a t-test applied to the individual time series yields far more significant cases than would be expected by chance (711 cases compared to roughly 70 that could be expected from chance alone). Further investigation of population characteristics leading to autoregression appeared desirable. I feel certain some constraints on the estimation of r_i would be desirable.

Interestingly, the cases in which significant r_i are estimated for individuals involve longer than average time series of information (line C, table 1). This finding might have been anticipated from the fact that the parameter estimates are more reliable when a longer series of information is available. What was not anticipated is that the average value for significant autoregressive coefficients was substantially negative. The autoregressive model was formulated on the hypothesis that a persistence of income position from year to year would lead to positive autocorrelation in the residuals from the trend line; the opposite was observed.

A simple explanation may be offered for the self-employed. Persons in self-employed occupations may make some arbitrary allocations of cost to accounting periods. Thus it is possible that a year of unusually high outlay resulting in exceptionally low income would be followed by another year of unusually low outlay and correspondingly higher profit. For non-self-employed occupations no obvious explanation for negative values of r_i presents itself. Because substantially more cases of significant r_i occur than would be expected on the basis of chance, further investigation of this phenomenon is required.

The principal conclusion to be drawn from Table 1 is that the autoregressive model fails to provide a superior explanation of the relative income variation of individuals than the simple trend model. Moreover ${}_ks_u'$ is larger than ${}_ks_u$ estimated for the simple trend model.[4] The latter finding implies that less unexplained differences among individuals occur when the simple trend model is used.

Study of birth groups did not reveal a subpopulation for which the autoregressive model was significant.

Classification of the population by principal occupation revealed two groups for whom the autoregressive parameters made a significant contribution to explained variance - self-employed businessmen and the relatively large group that includes semi-skilled and unskilled laborers. In both cases the mean r_i is negative; this supports our hypothesis for the self-employed, but we cannot offer an explanation for the laboring group. A summary of the model parameters appears in the lower portion of Table 1.

It is the case that roughly one-fifth of all cases contain a significant autoregressive component, and this component is substantially in excess of the proportion that could be expected on the basis of chance. Until some positive theory for the predominantly negative autoregression can be formulated and tested, it appeared wise to restrict attention to the simple trend model. That model gives a more satisfactory fit over the whole population; the autoregressive model cannot be directly applied in those cases where r_i is close to unity or inadmissible; the autoregressive model also produces a worse fit for the vast majority of individuals for whom no significant autoregressive component in the error term can be isolated.

4. Lifetime Income Patterns and Individual Income Variance

The estimated model of relative income position can be interpreted as a picture of the lifetime income experiences of different individuals. To generalize from the period of observation, 1948-1959, one must assume that relative income positions are determined by a typical pattern for a lifetime career, and are not influenced by the peculiarities of the labor market of the 1950's. In what follows we rely on that assumption and assemble lifetime estimates of ${}_k\alpha$, ${}_k\beta$, ${}_k\sigma_u$ and ${}_k\sigma_v$ for different occupational groups. In these charts the occupation last reported on a tax return determines the classification. The results suggest some typical career patterns and

and give a feeling for the importance of systematic stratification of the distribution as a factor producing increased variance of income among older persons.

Using the estimates to represent a lifetime pattern generalizes the usual cross-section hypothesis, namely that differences between individuals of different ages reflect a dynamic picture for a given individual with the passage of time. What we assume here is that the differentials obtained by studying a 5- to 12-year history for a single individual can be assembled for many individuals to give an accurage picture of the dynamics of income over a lifetime.

Since the data at hand reflect income reported for income tax purposes, neither young nor old persons are adequately represented. All of the 1930-34 cohort entered the labor force during our period of observation and to a large extent began filing tax returns at the time of entry. Those who received college training entered the labor force later than the bulk of their birth cohort. Thus many of the 1925-29 cohort and some of the 1915-24 dohort entered the labor force. A few of the members of the 1895-1904 cohort leave the labor force because of sickness or involuntary unemployment; most of the cohorts born prior to 1895 are retired. Filing of tax returns for the individuals born prior to 1895 is likely to indicate continued labor force attachments on more than a casual basis since few individuals have sufficient assets to require the filing of a tax return following retirement. (Mean earnings observed for this age cohort, 1947-1959, equalled \$3131, about three-fourths of the value for the sample as a whole). Care must be used in interpreting results for the youngest and oldest cohorts.

Figure 2 shows the expected relative income position of persons in different occupation groups as they grow older. Each point reflects the income position of workers in an occupation relative to all the members of their birth cohort. Professionals and managerial workers show substantial improvements in relative income position as they age. (A part of this trend may reflect the great shortage of professional workers during the 1950's rather than a "typical" career development.) A slight upward trend in the relative position of sales workers appears with increasing age, while other occupations exhibit more or less random variation about a fixed relative income position. The picture of Figure 2 indicates the manner in which the age-income profile observed in a cross-section should be modified for various occupational groups.

In Figure 3 values of ${}_k s_v$ are graphed for individuals according to their birth cohort and occupation in the last year observed. After entry into the labor force is completed, the values of ${}_k s_v$ show systematic increased for professional, managerial workers, and self-employed businessmen. Clerical, service, and semi- and unskilled workers exhibit no increases in standard error of estimate as older cohorts are observed. For many groups the variance of income declines again as they enter years in which members of their cohort retire. (The retired are excluded from the chart, even though they may have had employment during the period.)

We conclude that professional and managerial groups show a combination of improved relative income position and increased income variation over their lifetime. The increased income variation for the self-employed businessman is not associated with a systematic improvement in relative income position, while a large group of clerical, service, and semi- and unskilled workers show neither a trend in relative income position nor a change in within individual income variance relative to the mean for their dohort.

To complete the picture of careers we present estimates of ${}_k\sigma_u$ as a measure of the heterogeneity of individual experiences. Interpersonal variation association with the intercept shows the same stability for clerical, service, and semi- and unskilled workers as ${}_k s_v$. Whatever stratification, or layering, of the income distribution exists at the time of entry into these occupations persists throughout the lifetimes of these individuals. The professional and managerial workers show a radically different pattern. Interpersonal variation in the intercept increases substantially with age. (The extent of variation appears proportional to the value of ${}_k s_v$. As a consequence we conclude that the trends of individuals' income positions results in increasing layering or stratification of income with age.

This conclusion requires a re-examination of the model in [2]. The model must permit random variation in both trend and intercept over individuals. This is the random coefficients model (Swami [1970]). The extension of the model can be written as

$$ {}_k y_{it} = ({}_k\alpha + u_i) + ({}_k\beta + q_i)(t-1959) + {}_k v_{it} $$

where u_i, q_i are drawn from a bivariate normal distribution with a variance-covariance matrix

$$ {}_k\Sigma = \begin{pmatrix} {}_k\sigma^2_u & {}_k\mu \\ {}_k\mu & {}_k\sigma^2_q \end{pmatrix} \qquad [8] $$

The least squares technique used to estimate [2] provides unbiased estimators of ${}_k\alpha$, ${}_k\beta$. Estimates of ${}_k\Sigma$ are consistent, but not efficient (Swami [1970]).

In the context of [8] it is quite plausible that interpersonal variance about the intercept increases with age. That finding implies ${}_k\mu > 0$; that is, persons with relatively greater than average relative income positions within the k^{th} group also exhibit larger trends than the mean for the group.[5] ${}_k\mu > 0$ results in a "fanning" of income experiences with time, displayed in Figure 5. The interpersonal variance between the two individuals whose relative income experiences are shown in cc' and dd', will be greater at time 2 than at time 1. The positive deviation of u is associated with $q > 0$ so that the $\text{cov}(u,q) > 0$.

The increasing interpersonal variance ${}_k s_u$ for managers and professionals in different birth cohorts is corroborated by a weak, but positive correlation between a_i' and b_i' within these occupations.[6] The sample as a whole, shows no correlation between these parameters. To summarize in another way, the heterogeneity of trends in relative income position represents random movement of individuals with respect to their group intercept in the case of clerical, service, and semi- and unskilled workers. In the case of professional and managerial workers the trends persist systematically for particular individuals so that interpersonal variation rises as those individuals age.

5. Occupational Mobility and Income Variation

Study of occupational mobility gives still another insight into the variability of earnings and income experience of individuals. Table 3 presents estimates of the simple trend model for a variety of groups defined by observed changes in occupational status. The major occupation groups are the groups

- professional
- semi-professional
- managers
- self-employed businessmen
- farmers
- clerical workers
- sales
- service
- skilled workers
- semi- and unskilled workers

Several of these groups were further classified into detailed occupations; however, lack of precision in the self-reporting of blue-collar occupations prevented any detailed classification of those large occupational groupings. As a consequence the detailed occupations reflect fine distinctions within professional and managerial groups and crude differentiations (if any) among occupations included in the various categories.

In table 2, entry and departure from the labor force is treated as a change in major occupational group. In part A individuals who reported no change in major occupational group clearly show the most favorable trend in relative income position and the largest interpersonal variation by comparison with those individuals who reported some kind of change in occupational status. Individuals who reported more than one change in major occupational grouping clearly had the least favorable trend in relative income position and showed the least interpersonal variation. (Such persons must report at least three of the ten occupations listed above, or two such occupations and a change in labor force participation.) Among persons who reported one change in major occupation those who were in the new occupation a relatively short period of time reported more favorable income experiences. The meaning of this finding is confused by the fact that movement into or out of the labor force is counted as a shift in occupational group.

Table 2 employed clarifies the latter problem (also Table 10, Schroeder and David [1970]). Only persons who were employed in the labor force during the entire period for which they reported income for tax purposes are included. In this population the most favorable relative income trends are experienced by persons who have experienced a change in their detailed occupation, while the least favorable experience is again reported by persons with more than two occupational affiliations. Experiences of persons who remain employed and have two occupational affiliations are more favorable and show less interpersonal variation than those who enter or leave the labor force. (Compare row 2 all, with row C, employed).

Some insight into the meaning of these associations between relative income trends and occupation change can be gleaned from further classification of the groups in Table 2 by birth year cohorts. Our *a priori* hypotheses would be that occupation mobility would be likely to produce the most favorable change in income early in a career. Mobility between ages 55-64 is also likely to produce favorable impacts on income, given the manner in which the table is generated. Persons in the older age group who attempt a change in occupation and do not succeed will retire from the labor force altogether and will be excluded from the tabulation.

Unfortunately we cannot determine whether occupation change *causes* an improvement in relative income position as the model of the individual time series includes observations from years prior to the change and years following the change. Lack of an association in the expected direction would disprove our hypotheses, so that the data presented in Table 3 will indicate whether we should reject the null hypothesis. In fact, Table 3 indicates that persons born in 1925-34 who make a major shift in occupation have a superior income position in 1959 to those who do not. Interpersonal variation in those groups, as indicated by ${}_ks_u$ is also smaller. Finally, there does not appear to be a radical difference in the values of ${}_ks_v$ for mobile and non-mobile persons (considering the relatively small number of persons observed in some of the groups shown).

6. Conclusions

This descriptive lifetime history of income offers challenging subjects for analytical study.

a) The negative autoregression observed does not correspond to usual conceptions of the persistence of income positions.

b) The increased interpersonal variance (Figure 4) of the incomes of older professionals, managerial workers and the self-employed needs to be associated with a causal model related to skills and motivation.

c) The homogeneity of experiences of mobile workders needs to be related to the labor markets that they search, so that a mechanism causing the findings in Table 3 can be identified.

d) Lastly macroeconomic modeling should be undertaken to determine whether the relative income movements of professionals and managers during the 1950's is a peculiarity of the period investigated, or whether the lifetime income pattern is in some way associated with the evolution of their skills.

APPENDIX

Estimation of Parameters

The simple trend model was estimated by ordinary single equation least squares for each individual. However only those observations were used that met the information requirements of the autoregressive model. It might seem that by not using the additional values available for the simple trend model we have unnecessarily restricted ourselves to less efficient estimates of their parameters than what could easily be obtained. However since of major concern were the comparisons among the models that could be made we did not wish to confound these comparisons by utilizing the additional data.

The autoregressive model was estimated by the following transform (the subscript i has been suppressed from u_i, α_i, β_i, ρ_i and the data to simplify the notation):

$$y_t - \rho y_{t-1} = \alpha - \rho\alpha + \beta(t-1959) - \rho\beta(t-1-1959) + w_t - \rho w_{t-1} + (1-\rho)u \qquad \text{[A1]}$$

Rearranging terms and substituting v_t for $w_t - \rho w_{t-1}$ allows this to be put in the form

$$y_t = a_o + a_1 y_{t-1} + a_2(t-1959) \qquad \text{[A2]}$$

[A2] was estimated directly by ordinary least squares. a_i are defined by

$$a_1 = \rho$$
$$a_2 = (1-\rho)\beta \text{ and}$$
$$a_o = \rho\beta + (1-\rho)(\alpha + u) \qquad \text{[A3]}$$

so that we can solve for the original parameters by

$$\rho = a_1$$
$$\beta = a_2(1-a_1)^{-1} \text{ and}$$
$$\alpha + u = (1-a_1)^{-1}[a_o - a_1 a_2(1-a_1)^{-1}]. \qquad \text{[A4]}$$

Similarly the estimates of the a's in [A2] were inserted into [A4] to yield estimates of the parameters: $\hat{\alpha} + \hat{u}$, $\hat{\beta}$ and $\hat{\rho}$ in terms of $\hat{a}_o$, $\hat{a}_1$ and $\hat{a}_2$. The variances of the parameters were approximated using the variance-covariance matrix of the $\hat{a}$'s and the partial derivatives of the parameters with respect to the a's from [A4] in the usual Taylor series expansion (derivatives in terms of a's have been converted to their parameter equivalents):

$$\mathrm{Var}(\hat{\rho}) = \mathrm{Var}(\hat{a}_1),$$
$$\mathrm{Var}(\hat{\beta}) = (1-\hat{\rho})^{-2}[\hat{\beta}^2\mathrm{Var}(\hat{a}_1) + \mathrm{Var}(\hat{a}_2) + 2\hat{\beta}\mathrm{Cov}(\hat{a}_1\hat{a}_2)],$$
$$\begin{aligned}\mathrm{Var}(\hat{\alpha}+\hat{u}) = {} & (1-\hat{\rho})^{-2}\mathrm{Var}(\hat{a}_o) + [\hat{\alpha} + \hat{u} - \hat{\beta} \\ & - \hat{\rho}\hat{\beta}(1-\hat{\rho})^{-1}]^2\mathrm{Var}(\hat{a}_1) \\ & + \hat{\rho}^2(1-\hat{\rho})^{-2}\mathrm{Var}(\hat{a}_2) + 2(1-\hat{\rho})^{-1}[\hat{\alpha}+\hat{u}-\hat{\beta} \\ & - \hat{\rho}\hat{\beta}(1-\hat{\rho})^{-1}]^2\mathrm{Cov}(\hat{a}_o\hat{a}_1) \\ & - 2\hat{\rho}(1-\hat{\rho})^{-1}[\mathrm{Cov}(\hat{a}_o\hat{a}_2) \\ & + [\hat{\alpha} + \hat{u} - \hat{\beta} - \hat{\rho}\hat{\beta}(1-\hat{\rho})^{-1}]\mathrm{Cov}(a_1a_2)].\end{aligned}$$

These estimates were than aggregated according to Balestra and Nerlove [1966, 606-08] to obtain pooled parameters for the desired population.

FOOTNOTES

1. The birth year intervals used were: 1860-74, 1875-84, 1885-94, 1895-1904, 1905-14, 1915-24, 1925-29, 1930-34, 1935 and over. Adjusted gross income is defined by Wisconsin tax law and parallels the Federal definition. $C_t(B_i)$ was estimated from Census data on all income sources, so that the expected value of y_{it} is not necessarily unity.

2. An alternative model could be studied in which $r_i = {}_k r$ and a single autoregressive parameter is estimated for the k^{th} group. Inadmissible cases are unlikely with this alternative.

3. To test the significance of the autoregressive specification for the entire sample, we included the time series with inadmissible r_i. Thus identical populations are compared. As an alternative, the inadmissible series could be excluded from the pooled estimates for the simple trend model. That procedure subsamples the data on a rather arbitrary basis.

4. The value of ${}_k s_u = 1.329$ corresponds to the ${}_k s'_u = 1.558$ estimated in line C, Table 1, for the population with admissible r_i.

5. ${}_k\mu < 0$ may ultimately result in greater interpersonal variance, but there will be an intervening period where ${}_k\sigma_u$ is smaller than its initial value.

6. Actually the data are available for principle occupation, not last occupation reported.

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TABLE 1

Occupation, model	Number of Individuals	Degrees of Freedom Per Individual	Intercept Mean a	Intercept Std.Dev. $k^s u$	Mean Trend b	Error Std. Error of Estimate $k^s v$	Error Mean r
All							
A. Simple trend	3740	7.14	1.055	1.365	-0.0138	0.518	--
B. Autoregressive[a/]	3522	6.34	1.079	1.558	-0.0177	0.525	0.0154
C. Significant [b/]	711	7.44	1.180	1.255	-0.0129	0.435	-0.1274
D. Not significant	2811	6.06	1.054	1.625	-0.0190	0.551	0.0517
Self-employed businessmen							
A. Simple trend	281	7.55	1.068	1.844	-0.0265	0.804	--
B. Autoregressive[a/]	269	6.77	--	--	--	0.779	-0.015
C. Significant[b/]	47	n.a.	1.224	1.482	-0.0975	n.a.	-0.172
D. Not significant	222	n.a.	0.979	1.293	-0.0132	n.a.	0.019
Semi-skilled and unskilled							
A. Simple trend	1248	6.86	0.856	0.477	-0.0216	0.209	--
B. Autoregressive[a/]	1168	6.06	--	--	--	0.204	-0.018
C. Significant[b/]	237	n.a.	0.889	0.546	-0.0293	n.a.	-0.140
D. Not significant	931	n.a.	0.885	1.295	-0.0226	n.a.	0.050

a/ $-1 < r_i \leq 0.95$. The inadmissible cases have 3.06 degrees of freedom per individual.

b/ Probability of observing r_i when $\rho_i = 0$ is less than 0.02.

TABLE 2

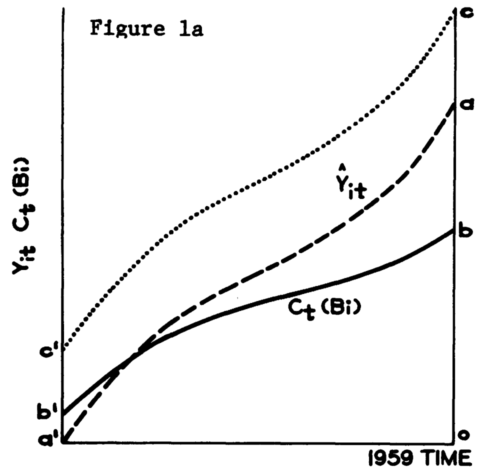
Category	Number	Intercept Value k^a	Intercept Std.Error $k^s u$	Trend k^b	Standard Error $k^s v$
All					
Unique major occupation	1920	1.134	1.562	-.0096	.589
Changed major occupation group	1820	0.970	1.115	-.0184	.428
More than once	562	0.874	0.645	-.0243	.370
Only once					
Less than 21% observations in last occupation	617	1.046	1.089	-.0143	.424
21-99% of observations in last occupation	641	0.989	1.419	-.0171	.479
Employed					
A. Unique detail occupation	1821	1.121	1.558	-.0108	.599
B. Unique major occupation, change in detailed occupation	71	1.157	0.752	.0014	.238
C. Two major occupations	801	1.050	0.821	-.0058	.371
D. Three or more occupations	204	0.918	0.486	-.0165	.381
E. All employed	(2897)	1.09	1.321	-.0095	.525

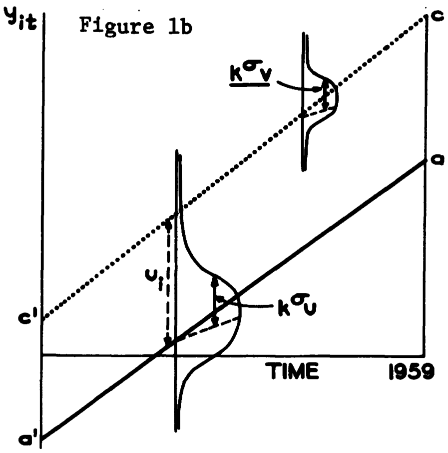
TABLE 3

Occupation observed 1947-1959		Birth Year 1860-1865	1885-1894	1895-1904	1905-1914	1915-1924	1925-1929	1930 1934
A.	k^a	1.01	1.24	1.27	1.09	0.99	0.89	0.89
B.		*	*	1.40	1.15	1.01	*	*
C.		*	1.33	1.09	1.01	0.98	0.95	1.08
D.		*	*	0.86	0.83	0.96	0.99	0.81
E.		1.09	1.27	1.22	1.05	0.99	0.92	0.96
A.	$k^s u$	1.38	2.20	1.84	1.40	0.79	0.50	0.76
B.		*	*	0.98	0.71	0.54	*	*
C.		*	1.79	1.10	0.70	0.65	0.49	0.40
D.		*	*	0.45	0.38	0.52	0.58	0.39
E.		1.30	2.09	1.66	1.20	0.72	0.51	0.61
A.	$k^s v$	0.75	0.68	0.57	0.35	0.25	0.24	0.31
B.		*	*	0.11	0.32	0.11	*	*
C.		*	0.37	0.63	0.32	0.25	0.24	0.31
D.		*	*	0.11	0.29	0.25	0.23	0.39
E.		0.72	0.24	0.57	0.33	0.25	0.22	0.32

*less than 10 observations.

A.- E. refer to the row stubs in Table 3.





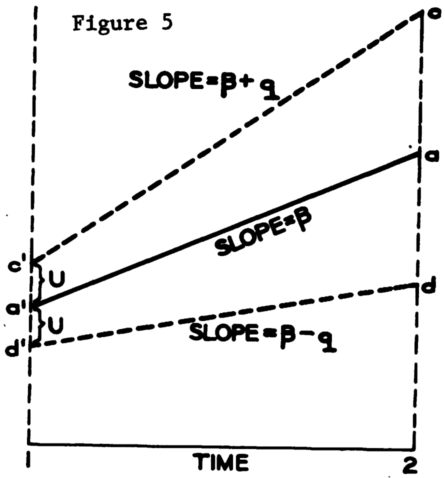


Figure 2

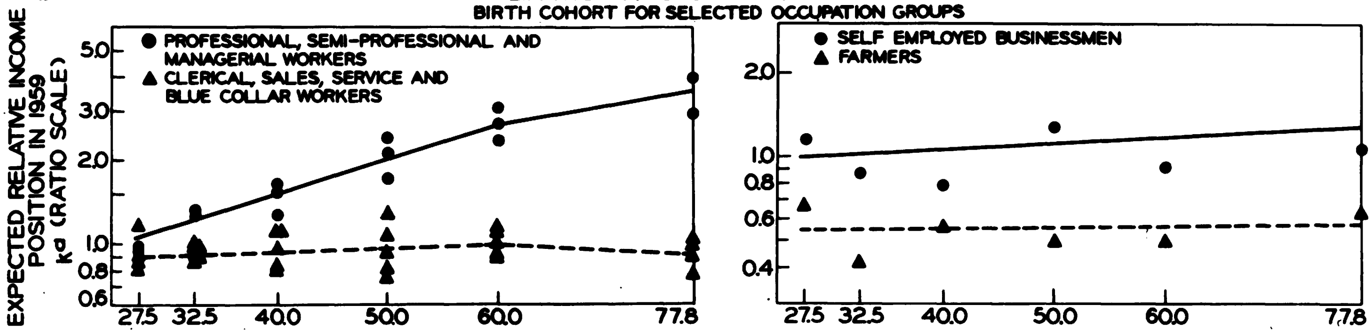


Figure 3

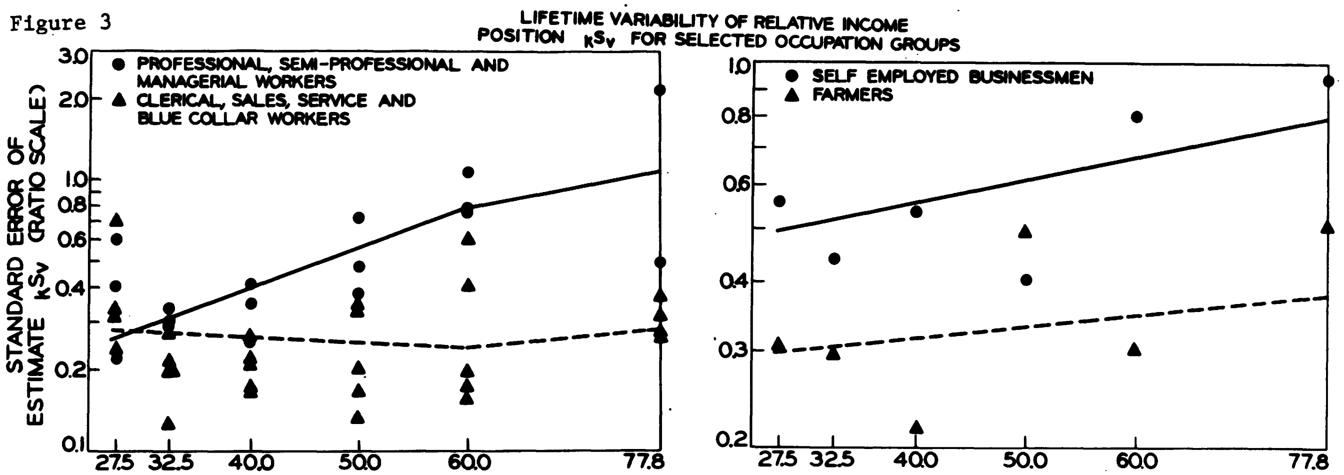


Figure 4